Free S_1^{ω}-algebras

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MV-algebras (with signature $(\oplus, \otimes, \neg, 0, 1)$ and type (2,2,1,0,0)) are the algebraic counterpart of the infinite valued Lukasiewicz sentential calculus [1], as Boolean algebras are with respect to the classical propositional logic. In contrast with what happens for Boolean algebras, there are *MV*-algebras which are not semi-simple, i.e. the intersection of their maximal ideals (the radical of *A*) is different from {0}. Non-zero elements from the radical of A are called infinitesimals and denoted by Rad(*A*).

The MV-algebra A is called perfect if $A = R^*(A) = \mathrm{Rad}(A) \cup \neg\,\mathrm{Rad}(A)$, where $\neg$ Rad(A) is the intersection of all maximal filters of *A*. The class of perfect *MV*-algebras does not form a variety and contains non-simple subdirectly irreducible *MV*-algebras. The variety $V(S_1^{\omega})$ is generated by all perfect *MV*-algebras is also generated by a single MV-chain S_1^{ω} (=*C*) (that is perfect) defined by Chang [1] . We name by S_1^{ω}-algebras all the algebras from the variety generated by S_1^{ω}.

$S_1^{\omega(0)} = \Gamma(Z, 1)$, $S_1^{\omega(1)} = S_1^{\omega} = C = \Gamma(Z \times_{lex} Z, (1, 0))$ with generator (0, 1) , $S_1^{\omega(m)} = \Gamma(Z \times_{lex} \dots \times_{lex} Z, (1, 0, \dots, 0))$ with generators (0, 0, … ,1), … , (0, 1, … , 0), where Γ is well known Mundici's functor [3] from the category of lattice ordered group with strong unit to the category of *MV*-algebras, the number of factors Z is equal to m+1, $m > 1$ and $\times_{lex}$ is the lexicographic product.

Theorem. *m-generated free* S_1^{ω}*-algebra* $F_{V(S_1\omega))}(m)$ *is isomorphic to* $(R^*((S_1^{\omega(m)})^{m!}))^{2m}$**.**

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References

[1] C. C. Chang, Algebraic Analysis of Many-Valued Logics, Trans. Amer. Math. Soc., 88(1958), 467-490.

[2] A. Di Nola, A. Lettieri, Perfect MV-algebras are categorically equivalent to Abelian "*l*-groups, Stud. Log. 88 (1994) 467490.

[3] D. Mundici, Interpretation of AF C -Algebras in Lukasiewicz Sentential Calculus, J. Funct. Analysis 65, (1986), 15-63.