SOME STRUCTURAL PROPERTIES OF $Q_{1,N}$- DEGREES

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Tennenbaum (see [3, p.159]) defined the notion of Q-reducibility on stes of natural numbers as follows: a set A is Q-*redusible* to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers), $x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B$.

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via a computable function f such that for all x, y,

$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset$ and $\bigcup_{x \epsilon \omega} W_{f(x)}$ is computable, then we say that A is $Q_{1,N}$ *-reducible* to B, and denoted $A \leq_{Q_{1,N}} B$. The notion of $Q_{1,N}$ -reducibility was introduced by Bulitko in [1].

A c.e. set M is r*-maximal* if $\overline{M}$ is infinite and for every computable R, either $R \cap \overline{M}$ or $\overline{R} \cap \overline{M}$ is finite.

Given computably enumerable (c.e.) sets $A \subseteq B$, A is a *major* subset of B if $B - A$ is infinite and for every c.e. set W,

$$\bar{B} \subseteq^* W \Rightarrow \bar{A} \subseteq^* W.$$

Lachlan [2] proved that each non-computable c.e. set has a major subset.

Our notation and terminology are standard and can be found in [3, 4].

Theorem 1. Let M be an r-maximal set, A be a major subset of M, B be an arbitrary set and $M - A \equiv_{Q_{1,N}} B$. Then $M - A \leq_m B$.

Theorem 2. The c.e. $Q_{1,N}$ -degrees are not dense.

Theorem 3. If A is an r-maximal set and B is a non-r-maximal hyperhypersimple set, then either $A|_{Q_{1,N}} B$, or there exist a non –r-maximal hyperpypersimple set C and an r-maximal set D such that

$$A <_{Q_{1,N}} D <_{Q_{1,N}} C <_{Q_{1,N}} B.$$

Theorem 4. Let A be a Σ_2^0 set, B be a c.e. set and $A \leq_Q B$. Then there exists a c.e. set C such that

$$A \leq_{Q_{1,N}} C \leq_Q B.$$

References

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[2] A.H.Lachlan, On the lattice of recursively enumerable sets. Trans. Amer. Math. Soc., 130 (1968),1-37.

[3] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.

[4]. R.I.Soare, Recursively Enumerable Sets and Degrees. Springer,1987.