Continuity of functional minimum of the nonlinear optimal problem and its applications

Tamaz Tadumadze

e-mail: tamaz.tadumadze@tsu.ge
Department of Mathematics, Iv. Javakhishvili Tbilisi State University, 13 University St., 0186 Tbilisi, Georgia

Let us consider the optimal problem

$$\dot{x} = f(t,x,u(t)),\ x \in R^n,\ t \in I = [t_0,t_1],\ u(\cdot) \in \Omega,\ x(t_0) = x_0, \tag{1}$$

$$J(u(\cdot)) = \int_{t_0}^{t_1} f^0(t,x(t),u(t))dt \to \min, \tag{2}$$

where $x(t) = x(t;u(\cdot))$ is the solution of problem (1) and Ω is the set of measurable control functions $u(t)$ with values in a compact U.

Theorem 1 ([1]). *For the problem (1)-(2) there exists an optimal control* $u_0(\cdot) \in \Omega$, *if the following conditions hold: 1) for each control* $u(\cdot) \in \Omega$ *the solution* $x(t;u(\cdot))$ *is defined on the interval* I *and the set* $\{x(t;u(\cdot)) : u(\cdot) \in \Omega\}$ *is bounded; 2) for each* $(t,x) \in I \times R^n$ *the set* $P_F(t,x) = \{(q^0,q) : \exists u \in U,$ $q^0 \geq f^0(t,x,u), q = f(t,x,u)\}$, *where* $F = (f^0, f)$, *is convex.*

Theorem 2. *Let the conditions of the Theorem 1 hold. Then for each* $\varepsilon > 0$ $\exists$ *a number* $\delta = \delta(\varepsilon) > 0$ *such that* for $\forall G_\varepsilon = (g_\varepsilon^0, g_\varepsilon)$ *satisfying the conditions: the set* P_{F+G_ε} *is convex and* $\sup\{ |G_\varepsilon(t,x,u)| : (t,x,u) \in I \times R^n \times U\} < \delta$, *for the perturbed optimal problem* $\dot{x} = f(t,x,u(t)) + g_\varepsilon(t,x,u(t)),\ x(t_0) = x_0$, $J(u(\cdot),\varepsilon) = \int_{t_0}^{t_1} [f^0(t,x(t),u(t)) + g_\varepsilon^0(t,x(t),u(t))]dt \to \min$ *there exists an optimal control* $u_\varepsilon(\cdot) \in \Omega$. *Besides,* $|J(u_\varepsilon(\cdot),\varepsilon) - J(u_0(\cdot))| < \varepsilon$.

On the basis of the Theorem 2, $u_\varepsilon(t)$ can be accepted as an approximation solution for the problem (1)-(2). For the illustration the non-smooth and singular optimal problems are considered. The Theorem 2 is proved by the scheme given in [2, 3]. The continuity of functional minimum for the optimization problems governed by ordinary and functional-differential equations are investigated in [2, 3].

References

[1] C. Olech, Existence theorems for optimal problems with vector-valued cost function. *Trans-Amer. Math. Soc.* **136** (1969), 159-180.
[2] T. Tadumadze , Some problems in the qualitative theory of optimal control.(Russian) *Tbilis. Gos. Univ.* , Tbilisi, 1983.
[3] T. Tadumadze , Continuity of the minimum of an integral functional in a nonlinear optimum control problem. *Differ. Equ.* **20** (6)(1984), 716-720 .